Ergodicity in statistical mechanics of interacting and disordered systems: Destroying and restoring equilibrium ergodic states

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Outline

1. Introduction - ergodicity and thermodynamic homogeneity
2. Ergodicty breaking in spin models
   - Models of interacting spins
   - Models with disorder and frustration - spin glasses
3. Real-replica method for disordered frustrated systems
   - Replication of the phase space
   - Discrete replica-symmetry (replica-independence) breaking
   - Continuous replica-symmetry breaking
4. Application: Solvable cases
   - Replica symmetric and one-level RSB -- Ising
   - Infinite RSB - asymptotic solution -- Ising
   - Potts and p-spin glass
5. Conclusions

Microscopic dynamics of large systems

Instantaneous microscopic dynamics of large systems

- Macroscopic objects -- aggregate of microscopic elements
- Superposition principle -- large system of coupled Hamilton equations determining the phase-space trajectory
- Impossibility to determine complete initial conditions
- Inability to determine the actual trajectory of large microscopic states
- Macroscopic state: phase space fluid with Liouville equation
- Entropy as a measure of macroscopic uncertainty

How do we determine macroscopic properties without solving Liouville equation?

Macroscopic time scales

Large time scales - macroscopic stationarity

- Macroscopic measurements -- on large time scales (relaxation time)
- Time fluctuations on microscopic time scales macroscopically unimportant
- Only time averaged quantities measurable (relevant)
- Energy as the only relevant macroscopically conserved quantity restriction on the phase-space trajectory
- Thermodynamic equilibrium -- macroscopically static state

How do we calculate time averaged quantities?

Ergodicity in equilibrium statistical physics

- Fundamental ergodic theorem (Birkhoff)

$$\langle f \rangle_T \equiv \lim_{T\to\infty} \frac{1}{T} \int_{t_0}^{t_0+T} f(X(t))dt = \frac{1}{\Sigma_E} \int_{S_E} f(X) dS_E \equiv \langle f \rangle_S$$

- Phase space homogeneously covered by the phase trajectory

$$\lim_{T\to\infty} \frac{\tau_R}{T} = \frac{\Sigma_R(E)}{\Sigma(E)}$$

- Equilibrium ergodic macroscopic state
  - homogeneously spread over the allowed phase space
  - characterized by homogeneous parameters ($\{E, T\}$, $\{N, \mu\}$,...)
  - number of relevant parameters (Legendre pairs) á priori unknown

How do we determine the phase space covered by the phase space trajectory?

Homogeneity of thermodynamic potentials

- Homogeneity in the phase space

$$S(E) = k_B \ln \Gamma(E) = \frac{k_B}{\nu} \ln \Gamma(E)^{\nu} = \frac{k_B}{\nu} \ln \Gamma(\nu E)$$

$$F(T) = -\ \frac{k_B T}{\nu} \ln \left[\mathrm{Tr}\, e^{-\beta H} \right]^{\nu} = -\ \frac{k_B T}{\nu} \ln \left[\mathrm{Tr}\, e^{-\beta \nu H} \right]$$

- Homogeneity of thermodynamic potentials (Euler)

$$\alpha\, F(T, V, N, \ldots, X_i, \ldots) = F(T, \alpha V, \alpha N, \ldots, \alpha X_i, \ldots)$$

Density of the free energy $f = F/N$
-- function of only densities of extensive variables X_i/N

Ergodicity (homogeneity) guarantees existence and uniqueness of the thermodynamic limit $N \to \infty$

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Ergodicity breaking

- Ergodicity gives meaning to statistical averages
- Thermodynamic properties in the infinite-volume limit
- Ergodicity breaking -- improper statistical phase space
  1. caused by a phase transition breaking a symmetry of the Hamiltonian
  2. without apparent symmetry breaking -- glass-like behavior
- Means to restore ergodicity
  1. Measurable (physical) symmetry breaking fields
  2. Real replicas (non-measurable symmetry breaking fields)

Ergodicity must be restored to establish stable equilibrium